Probability Axioms

In defining a probability measure, we require that it satisfies the following axioms:

1. $0 \leq P(A) \leq 1$ for all A in event space
2. $P(\phi) = 0$
3. $P(S) = 1$
4. (Countable Additivity) $P(A_1 \text{ or } A_2 \text{ or } A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$ where A_n is a sequence of disjoints event.

Random Variable

<u>Definition version 1</u>: A random variable is a function from the sample space S to the real number line R.

<u>Definition version 2</u>: A random variable is a numeric value based on the outcome of a random event.

Random variables provide a condensed way of looking at problems. If we know the probabilistic behaviour of each X, we can solve the problem using less effort than combinations.

Example 1: *Recall from last week, consider flipping a coin once and record its face value.*

1. Sample space is S = {Head, Tail}.

2. A collection of all possible events are {φ, Head, Tail, {Head, Tail}}.

3. The probability measure P assigns a real number that represents "likeness" of each event in the event space.

P(Head) = P(Tail) = ½

P({Head, Tail}) = P(either a head or a tail shows up) = 1

P(φ) = P(neither a head nor a tail shows up) = 0

How can we express the above problem using a random variable?

- Let X model the outcomes of a coin toss experiment.

  $$X = \begin{cases} 1, & if\ head \\ 0, & if\ tail \end{cases}$$

  Probability distribution of X:

  $P(X = 1) = ½$

  $P(X = 0) = ½$

P(S) = P(X = 1) + P(X = 0) =1

P(ϕ) = 0

- Another way to model the outcomes of a coin flip:

  $X = \begin{cases} 100, & if\ head \\ 200, & if\ tail \end{cases}$

  Probability distribution of X:

  P(X = 100) = ½

  P(X = 200) = ½

  P(S) = P(X = 100) + P(X = 200) =1

  P(ϕ) = 0

Question: Is this probability distribution valid? Check axioms above!

Binomial Model

- Consider flipping n coins, each of which has (independent) probability ***p*** of coming up heads, and probability ***1 –p*** of coming up tails. (Again 0 < ***p*** < 1.)
  [two possible outcomes – success or failure; each flip follows Bernoulli Distribution]

- Let X(random variable) be the total number of heads showing. We see that X can take values 0, 1, 2,…, n, where X = 0 means getting no head out of n flips, X = 1 means getting 1 head out of n flips.

- In general, X = number of successes in n trials. And the random variable X is said to have the Binomial(n, p). We write this as X ~ Binomial (n, p).

Question 1: The random variable X has this distribution

Value of X	10	8
Probability	0.7	0.3

a) Compute the mean of X
b) Compute the variance and standard deviation of X

Question 2:
A particular tennis player makes a successful first serve 40% of the time. Assume that each serve is independent of the others. If she serves 6 times, what is the probability that she gets

a) All six serves in?
b) Exactly four serves in?
c) At least 4 serves in?
d) No more than 4 serves in?