Probability Model(or triple)

1. A sample space (*S*) containing all possible outcomes of an experiment
2. A set of all possible events (all possible subsets of *S;* event space)
3. A probability measure defined on the event space

Example: Consider flipping a coin once and record its face value.

1. Sample space is *S* = {Head, Tail}.
2. A collection of all possible events are {ϕ, Head, Tail, {Head, Tail}}.
3. The probability measure P assigns a real number that represents "likeness" of each event in the event space.

P(Head) = P(Tail) = ½

P({Head, Tail}) = P(either a head or a tail shows up) = 1

P(ϕ) = P(neither a head nor a tail shows up) = 0

Properties of Probability Models

1. Complement Rule

   P(not A) = $P(A^C)$ = 1 – P(A)

2. Addition Rule (General Case)

   P(A or B) = P(A) + P(B) – P(A and B) for any two events, disjoint or not.

   Disjoint: the probability of A and B happening at the same time is 0

3. Conditional Probability

   "Probability of B given A" = P(B|A) = $\frac{\text{P(A and B)}}{\text{P(A)}}$

4. Independence

   Events A and B are independent iff the probability of B is the same when we are given that A has occurred.

   P(B|A) = P(B)

5. General Multiplication Rule

   P(A and B) = P(A) x P(B|A)

   If A and B are independent, P(A and B) = P(A) x P(B)

Question 1: (exercise 30 in Chapter 11 on Page 417)

In 2014, 52.3% of all immigrants to Canada were females, 18.6% were under 18 years old, and 9.0% were females under 18 years old. Find the probability that a randomly selected person who immigrated to Canada in 2014 was

a) Females and at least 18 years old
b) Either female or under 18 years old
c) Male and at least 18 years old

Question 2: (exercise 21 in Chapter 12 with a little twist)

You roll a fair die for three times. What is the probability that

a) You roll all 6s?
b) You roll at least one 5?
c) You roll a 6 on the third row given that the first two show 5? Is this result surprising?