Solution

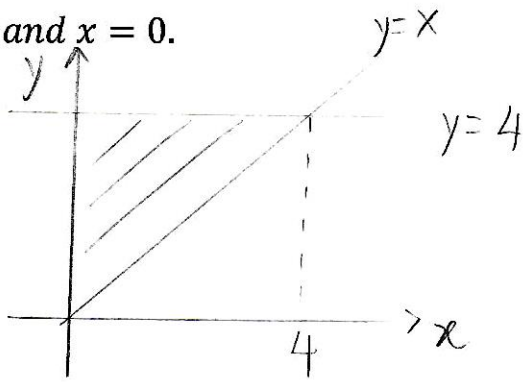
MATB41 Quiz 7 Tutorial 5

Last Name: _____ First Name: _____

Student Number: _____

Set up iterated integrals of both orders of integration. Then evaluate the double integral using the easier order and explain why it is easier.

$$\iint_D y^2 e^{xy}\,dA, D \text{ is bounded by } y = x, y = 4, \text{and } x = 0.$$



① $\int_0^4 \int_0^y y^2 e^{xy}\,dx\,dy$

② $\int_0^4 \int_x^4 y^2 e^{xy}\,dx\,dy$

② is more difficult to integrate than ① because ② involves multiplication of y^2 and e^{xy}.

① $\int_0^4 \int_0^y y^2 e^{xy}\,dx\,dy$

$= \int_0^4 y e^{xy} \Big|_0^y dy$

$= \int_0^4 y e^{y^2} - y\,dy$

$= \frac{1}{2} e^{y^2} - \frac{y^2}{2} \Big|_0^4$

$= \frac{1}{2} e^{16} - \frac{16}{2} - \frac{1}{2}$

$= \frac{1}{2}(e^{16} - 17)$

Solution

MATB41 Quiz 8 Tutorial 9

Last Name: _____ First Name: _____

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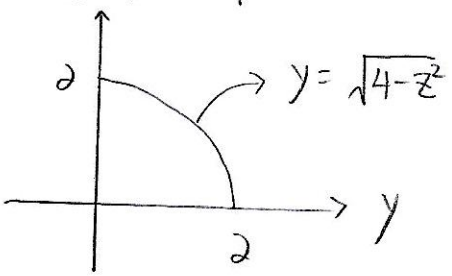
Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y,\ x = 0,\ z = 0$ in the first octant.

Project onto yz-plane



$$\int_0^2 \int_0^{\sqrt{4-z^2}} \int_0^{2y} 1 \, dx \, dy \, dz$$

$$= \int_0^2 \int_0^{\sqrt{4-z^2}} 2y \, dy \, dz$$

$$= \int_0^2 y^2 \Big|_0^{\sqrt{4-z^2}} dz$$

$$= \int_0^2 4 - z^2 \, dz$$

$$= 4z - \frac{z^3}{3} \Big|_0^2$$

$$= 8 - \frac{8}{3}$$

$$= \frac{16}{3}$$

Solution.

MATB41 Quiz 9 Tutorial 5

Last Name: _____ First Name: _____

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Express the integral $\iiint_E f(x,y,z)dV$ as an iterated integral in 4 different ways in the order as specified below, where E is the solid bounded by the given surfaces $y = 4 - x^2 - z^2, y = 0$.

Please complete the boundaries of the integrals below.

1. $\iiint_E f(x,y,z)dxdydz = \int_{-2}^{2}\int_{0}^{4-z^2}\int_{-\sqrt{4-z^2-y}}^{\sqrt{4-z^2-y}} f(x,y,z)\,dx\,dy\,dz$

2. $\iiint_E f(x,y,z)dzdydx = \int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2-y}}^{\sqrt{4-x^2-y}} f(x,y,z)\,dz\,dy\,dx.$

3. $\iiint_E f(x,y,z)dxdzdy = \int_{0}^{4}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{-\sqrt{4-z^2-y}}^{\sqrt{4-z^2-y}} f(x,y,z)\,dx\,dz\,dy$

4. $\iiint_E f(x,y,z)dydzdx$

$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{4-x^2-z^2} f(x,y,z)\,dy\,dz\,dx.$

Solution.

MATB41 Quiz 10 Tutorial 5

Last Name: _____ First Name: _____

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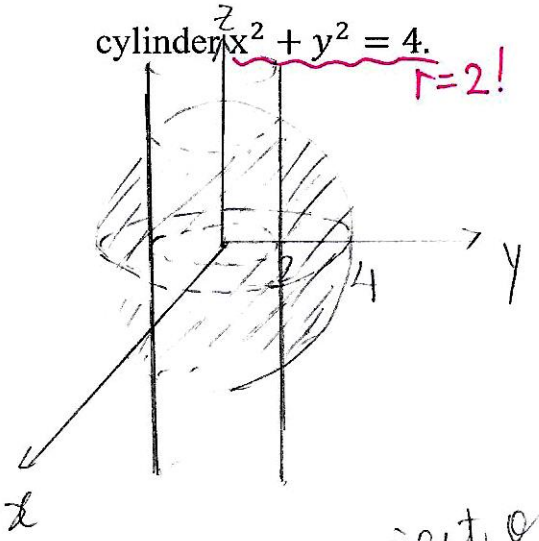
Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

$\rho = 4$!

$r = 2$!



Sol'n:

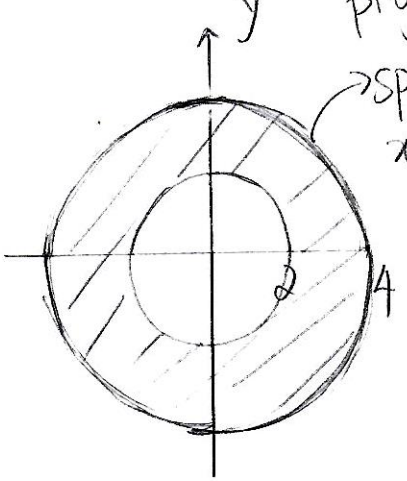
Switch to polar coordinate on xy-plane

$x^2+y^2+z^2 = 16$

$r^2+z^2 = 16$

$z = \pm\sqrt{16-r^2}$

$-\sqrt{16-r^2} \le z \le \sqrt{16-r^2}$



project of sphere on xy-plane.
$x^2+y^2+z^2=16$.

$$\int_0^{2\pi}\int_2^4\int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_2^4 rz\,\Big|_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}}\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_2^4 2r\cdot\sqrt{16-r^2}\,dr\,d\theta$$

Aside:

$u = 16-r^2$

$\frac{du}{dr} = -2r$

$dr = \frac{du}{-2r}$

$2 \le r \le 4$

$12 \le u \le 0$

$$= \int_0^{2\pi}\int_{12}^0 2r\cdot\sqrt{u}\,\frac{du}{-2r}\,du\,d\theta$$

$$= \int_0^{2\pi}\int_0^{12}\sqrt{u}\,du\,d\theta$$

$$= \int_0^{2\pi}\frac{u^{3/2}}{(\frac{3}{2})}\,\Big|_0^{12}\,d\theta$$

$$= \int_0^{2\pi} 16\sqrt{3}\, d\theta$$

$$= 16\sqrt{3}\,\theta \Big|_0^{2\pi}$$

$$= 32\sqrt{3}\,\pi$$