Tutorial (10)

Example 1:

Compute the volume of the solid bounded by the surface $z = 3 - x^2 - y^2$ the planes $x = 1$, $x = 0$, $y = 0$, and $y = 2$ and the xy-plane.

Sol'n:

<u>Note:</u> $\iint_D f(x,y)\, dA$ of positive $f(x,y)$ can be interpreted as the volume under the surface $z = f(x,y)$ over the region D.

$\iint_D f(x,y)\, dA$ is the volume between the surface $z = f(x,y)$ and the xy-plane.

$$\int_{x=0}^{x=1} \int_{y=0}^{y=2} 3 - x^2 - y^2 \; dy\, dx.$$

$$= \int_{x=0}^{x=1} 3y - x^2 y - y^3/3 \Big|_{y=0}^{y=2} dx$$

$$= \int_{x=0}^{x=1} 6 - 2x^2 - \frac{8}{3} \; dx$$

$$= \int_{x=0}^{x=1} -2x^2 + \frac{10}{3} \; dx$$

$$= -\frac{2x^3}{3} + \frac{10}{3}x \Big|_0^1$$

$$= \frac{8}{3}$$

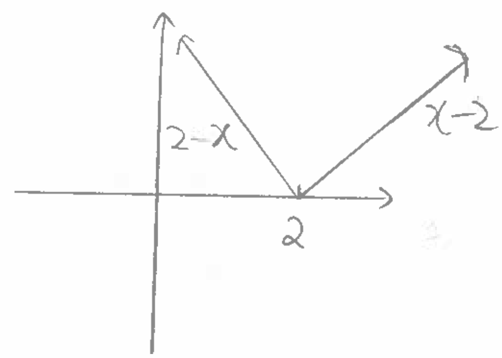
1(a) Evaluate $\iint_D |x-2| \cos^2 y$, $D = \underbrace{[1,4]}_{x} \times \underbrace{[-\pi,0]}_{y}$.

$$\int_1^4 |x-2| \int_{-\pi}^{0} \cos^2 y \;\, dy\, dx$$

$$= \underbrace{\int_1^4 |x-2|\, dx}_{①} \cdot \underbrace{\int_{-\pi}^{0} \cos^2 y\, dy}_{②}$$

① :

$$|x-2| = \begin{cases} x-2, & x \geqslant 2 \\ 2-x, & x < 2 \end{cases}$$



$$\int_1^4 |x-2|\; dx$$

$$= \int_1^2 2-x \;\, dx \;+ \int_2^4 x-2\; dx$$

$$= \left[2x - \frac{x^2}{2}\Big|_1^2\right] + \left[\frac{x^2}{2} - 2x\Big|_2^4\right] \dots$$

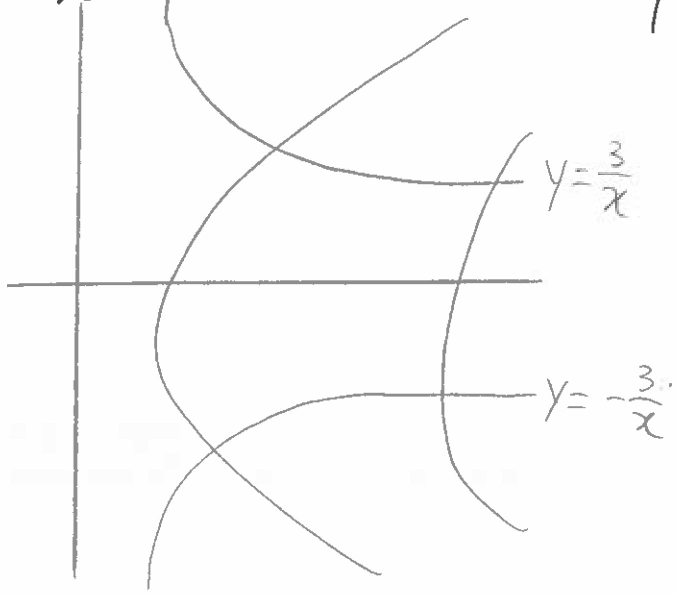
$$= [4-2-2+\tfrac{1}{2}] + [8-8-2-2]$$

$$=$$

② : $\displaystyle\int_{-\pi}^{0} \cos^2 y \;\, dy = \int_{-\pi}^{0} \left(\tfrac{1}{2} + \tfrac{1}{2}\cos(2y)\right)$

Example 2 :

Evaluate $\int_D (x^2+y^2)\cos(xy)\, dA$ where D is the region to the right of the y-axis that is bounded by the hyperbolas $y=\frac{3}{x}$, $y=\frac{-3}{x}$, $x^2-y^2=1$ and $x^2-y^2=9$.



Let $u=xy$, $v=x^2-y^2$

$$\begin{cases} -3 \le u \le 3 \\ 1 \le v \le 9 \end{cases}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{pmatrix} y & x \\ 2x & -2y \end{pmatrix}$$

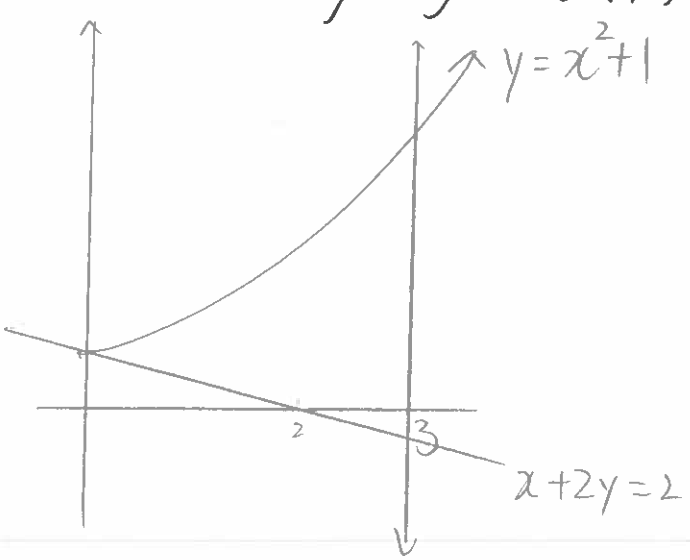
$$= -2(x^2+y^2)$$

$$Jg = -\frac{1}{2(x^2+y^2)} \quad \int_{-3}^{3}\int_1^9 \cos(v)\, dv\, du$$

$$\int_{-3}^{3}\int_1^9 (x^2+y^2)\cos(xy)\, |Jg|\, dv\, du$$

Example 3 : 1b

Evaluate $\iint_D y^2\, dA$ where D is the region bounded by $y=x^2+1$, $x+2y=2$ and $x=3$.



$$\int_{x=0}^{x=3}\int_{y=1-\frac{x}{2}}^{y=x^2+1} y^2\, dy\, dx$$

$$= \int_{x=0}^{x=3} \frac{1}{3}y^3 \Big|_{y=1-\frac{x}{2}}^{y=x^2+1} dx$$

$$= \int_{x=0}^{x=3} \frac{1}{3}\left[(x^2+1)^3\right] - \frac{1}{3}\left[\left(1-\frac{x}{2}\right)^3\right] dx$$

...

I will leave the rest to you ...